György Jóna: How to gauge structure of a real network?[1]

Abstract

The paper empirically elaborates and empirically tests a new complex model of network structure exploration (CMNSE). This model measures simultaneously structural features of nodes and edges of a real network to understand traits, functions and robustness of complex systems (networks). Papers have so far regularly focused on dissecting structural properties of links by applying degree distribution meanwhile structural traits of vertices received scant attention. To fill this gap, CMNSE quantifies both structural attributes of links and vertices together. On one hand, structural features of edges are measured traditionally with degree distribution and, on the other hand, structural traits of nodes are gauged with quadrat analysis. Topology of the newest form of economic network (wine shops' coopetitive network) is also tested by CMNSE. Empirical findings reveal that this coopetitive network does not have scale-free property since its degree exponent belongs to the anomalous regime. Moreover, nodes are clustered significantly in the periphery of networked space and the hub is localized relatively far from them in the centre of network. CMNSE perceives the presence of giant component and pinpoints the spatial position of the hub and vertices so as to real networks could be developed or attacked in the future.

Keywords

network structure, degree distribution, quadrat analysis, spatial approach, real networks

1.Introduction

The paper develops and empirically tests a novel complex model of network structure exploration (CMNSE) that can analyze structures of real networks in a multifaceted way. Obviously, the network topology must be scrutinized to understand functions, interior features, robustness, vulnerability, accomplishment, effects and externalities of complex systems (Barabási 2016, Chagnon et al 2016, Newman 2000). Only the architecture of *real* networks is examined by CMNSE. The notion of real network refers tored a type of network wherein vertices are not randomly linked to each other and that can be found in the real world as well; the real networks cannot be described with Erdős-Rényi random graph theory (Bollás

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2001). Furthermore, the notion of network structure here indicates that nodes and edges are organized and arranged in the networked space.

The paper is constructed as follows. The section theoretical underpinnings reviews previous and the most relevant empirical results of network morphology. After every detail of CMNSE is demonstrated in section Method, the model is tested empirically on a Hungarian network of wine shops (NWS) in section Results. Finally, section Conclusion summaries the main points of CMNSE, empirical findings and the future research directions as well.

2.Theoretical background and empirical overviews

Network structure has been studied regularly by degree distribution (i.e. how many linkages belong to vertices in a certain network). For example, topology of biological (Barabási 2016, Barabási-Oltvai 2005, Kveler etal. 2018, Santolini-Barabási 2018, Shen etal 2015), medical (Barabási 2016, Barabási-Oltvai 2005, Csermely – Korcsmáros – Kiss 2013b), ecological (Melián etal. 2009, Palla etal. 2005, Poisot-Gravel 2014), societal (Barabási etal. 2002, Glowacki etal. 2016, Jackson 2016, Kossinets-Watts 2006, Palla etal. 2005), economic (Anand-Craig-Goetz 2014, Goyal 2007, Jackson 2016, Knieps 2015, Zhang-Du 2017) and informational (Jeong-Albert-Barabási 1999, Gleeson etal 2016, Stopczynski-Pentland-Lehmann 2018) networks have so far been characterized with degree distribution. Csermely et al. (2013a, 2013b) classify accurately types of structural properties of linkages. Besides, some multidimensional studies, however, combine several network indices to measure network scheme. Such as, Uchida-Shirayama (2008) inspect a network system by employing at the same time degree distribution, clustering coefficients, short average path length and degree correlation. Hagberg-Swart-Schult (2008) also integrate network diameter, betweenness centrality, shortest path, degree distribution and clustering coefficient in order to describe properties of a network structure. Chagnon et al. (2016) explore ecological network topology by synthesizing six network metrics: the C-score index, nestedness, betweenness centrality, modularity, power-law fit to degree distribution and interaction strength asymmetry. Russel et al. (2017) employ simultaneously closeness centrality, betweenness centrality and clustering coefficient so as to design network structure of children' societal environment engaged in post-traumatic stress. Apparently, empirical network measurements have so far applied usually degree distribution and hardly ever multilayer approaches.

Notwithstanding, a few empirical inquiries have so far been analyzed structural features of nodes structure applying heuristic measure-based methods or probabilistic inference-based model (Chai etal 2013, Chen etal 2016, Duan etal. 2013). All of these overlooked to explore structural traits of edges. Structural characteristics of vertices and edges have so far been scrutinized separately and not together.

Of course, above mentioned methods are referred to as adequately and useful method but is not enough to unfold entirely network topology. Whereas a real network subsumes vertices and edges N=(V,E) therefore both of structural traits must be investigated simultaneously to map up network architecture in a complex way. This complex analytical approach cannot be omitted in scientific research.

Simply put,

Measuring of structure of a real network =

structural features of *edges*

+

structural features of *nodes*.

In the paper, CMNSE is developed that perceives and quantifies simultaneously both of structural properties of vertices *and* links together. The next section describes and explains CMNSE.

3.The network model

Two components of CMNSE can be distinguished. The first one inspects structural features of links by degree distribution. The second one dissects the structural characteristics of nodes by quadrat analysis. Apparently, a real network emerges as a complex system thus it must be studied in a complex way. Firstly, the paper concentrates on how structural properties of linkages are measured.

3.1. Measuring structural features of edges

A real network (*N*) embraces a finite, nonempty set $V = \{v_i\}$ of vertices (*V*) and a finite, nonempty set $E = \{e_j\}$ of edges (*E*). The number of vertices in *V* is defined as *i*, and the number of edges in *E* is referred to as *j*. Since CMNSE builds on previous scientific inquiries, structural feature of edges is gauged consequently with degree distribution. Degree distribution p_k can be estimated well:

$$p_k \sim C b^{-\gamma} \tag{1}$$

where *C* is a constant, *b* means a variable and $-\gamma$ expresses degree exponent. The value of $-\gamma$ shows structural characteristics of network connections, the CMNSE, as a result, concentrates on the value of degree exponent (Barabási 2016, Goh-Kahng-Kim 2001).

3.2. Measuring structural features of nodes

Gauging of structural patterns of nodes might be more difficult than degree distribution. The starting point of CMNSE is that every real network possesses spatial extension, size, form and dimension. For example, the geographical distances among vertices are relatively long in the motorway network (Adamatzky et al. 2017), in networks for commodities delivery (Barthélemy 2017), in river network (Rodrigue-Ronaldo 1997), in power grid network (Kim et al. 2018) and in street network (Gil 2016) as well. Shorter physical distances can be found regularly in social networks (Latour 2011) or in the networks of small-and-medium-sized enterprises (Balister etal. 2018, Törnroos et al. 2017). Nevertheless, the physical distances among vertices can be measured in centimeters or millimeters in underground hyphal network (Friese-Allen 1991), in the three-dimensional integrated circuits (Wong 2007), in neurons' network in the brain (Dehmamy-Milanlouei-Barabási 2018), in circulatory network systems (West-Brown 2003) or in other cell networks (Gartner-Prescher-Lavis 2017) as well. Obviously, all of real networks have spatial extension irrespective of their sizes, ages or types. Spatial characteristics of real networks permit to describe and to capture structural traits of vertices, the spatial position of hubs and nodes, and physical interplay among them. By taking into account the spatial distribution of vertices, we can answer the question where network agents are clustering or thinning out in the networked space. Nodes of spatial distribution are measured with quadrat analysis (i.e. it focuses on spatial patterns and allocation of nodes by comparing the number of vertices among the cells; the sizes of a grid have no mathematical rules or theorems, it is defined by always the researcher) (Brinkhoff – Kresse 2012, Jinghu-Junfeng-Yibo 2015, Reginald 1977). The spatial distribution of nodes is analyzed empirically by quadrat analysis because it is regarded as a useful, simple, elegant and reliable method (Robinson et al 2016).

Firstly the networked space has to be delimited geographically. To this end, the outermost vertices are connected to each other obtaining the physical boundary of networked space. The outermost node of the network is defined in this paper as a vertex that geographically, cartographically locates the outermost of the networked space.

Subsequently, grids are superimposed over the spatial layout of the networked space and the number of events falling in each grid area is counted. The results of quadrat analysis are characterized by the variance-to-mean ratio (VMR) test (O'Sullivan-David 2010, Robinson et al 2016). To implement VMR, mean grid count (μ) has to be calculated:

$$\mu = \frac{V}{x}$$

(2)

where x expresses the number of quadrats.

After this, $x(a-\mu)^2$ is computed where a means the number of events. The variance (s^2) is obtained:

$$s^2 = \frac{1}{V}\sum_{i=1}^{v}(a-\mu)^2$$

(3)

and

$$VMR = \frac{s^2}{\mu}$$

(4)

If VMR<1, the variance is low, regularly is zero, the distribution of nodes is uniform. When the spatial allocation of points is random/stochastic (i. e. follows Poisson distribution pattern) then VMR=1, namely the mean and variance are equal. If VMR>1 (variance is greater than mean), distribution is clustered. In a nutshell, point distribution could be clustered (attracting), stochastic/random (Poisson) and uniform (repelling) (Robinson et al 2016).

To put it short, the structural features of edges are gauged with degree distribution and the structural features of nodes are measured by quadrat analysis. Firstly degree distribution should be calculated to obtain structural traits of links. After this, structural attributes of edges have to be defined. To implement it, spatial boundaries of a real network have to be delimited. Later, quadrats are superimposed over the map of networked space and spatial patterns of points thus are analyzed with VMR. CMNSE synthesizes results of degree distribution and quadrat analysis to study network structure in complex way.

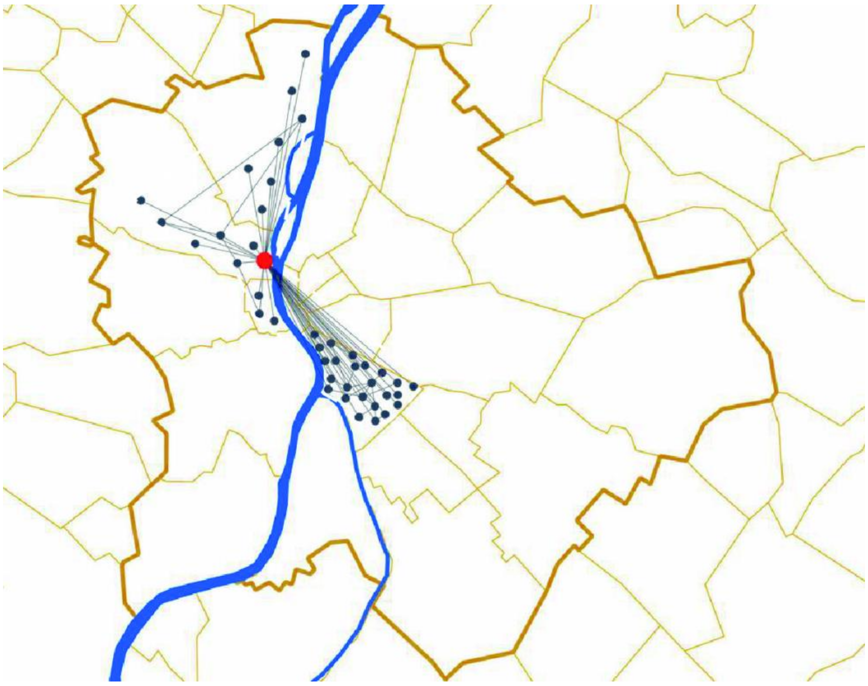
In the next section, a structure of a new type of economic network will be examined empirically by testing CMNSE.

4. Results and discussion

4.1. General features of new form of economic network

In this section, the topology of a totally new type of economic network, called coopetitive network (i. e. dynamic inter-firm relationship in which rivals compete and collaborate with each other simultaneously in some business fields so as to realize higher profit rate), is studied with CMNSE. More specifically, in 2011 some Hungarian owners of wine shops constructed informally a coopetitive network in Budapest to reduce purchase prices together and increase their complex accomplishments; spatial layout of network of wine shops (NWS) is depicted in Figure 1.

Figure 1. Spatial layout of coopetitive network of wine shops in Budapest



Resource: my edition.

NWS solely includes microenterprises and small and medium-sized entrepreneurs. Coopeting partners cooperate in only two business activities, namely they mutually purchase and transport bottles of wines. Network agents purchase in a bulk from wineries to receive discounts and transport products together in order to decrease expenditures and to increase, as a result, profit. However, rivals compete with each other for more consumers, for well-qualified employers, for innovations, for relational capital, for recipes of special foods, for reliable accountants, to name just a few. Similar coopetitive networks may be found in Eastern European regions as well (Jóna-Tóth 2017, Jóna 2018).

In NWS a node demonstrates the physical location of a firm and an edge means undirected and unweighted coopetitive interactions (emails, phone communications, face-to-face conversations, etc.) among rivals. Coopetitive activities (mutual dispatch and purchasing) are planned, managed and coordinated by emails hence the linkage of length between rivals is defined as the Euclidean distance (non-spatial relationships); it is illustrated as crow flies in Figure 1. In brief, NWS is referred to as a bottom-up real network wherein loops and isolated nodes ($k_i = 0$) cannot be found (k_i expresses the node degree). NWS functionalizes as an informal network without any formal name list, sociological snowball method, therefore, was used to map up the whole network and to muster raw network dataset (Heckathorn – Cameron 2017).

4.2. Testing complex model of network structure exploration (CMNSE)

Firstly, the degree distribution of NWS is measured. After this, results of quadrat analysis are characterized with VMR. Finally, both findings are interpreted together to describe multilevel way of topology of real network.

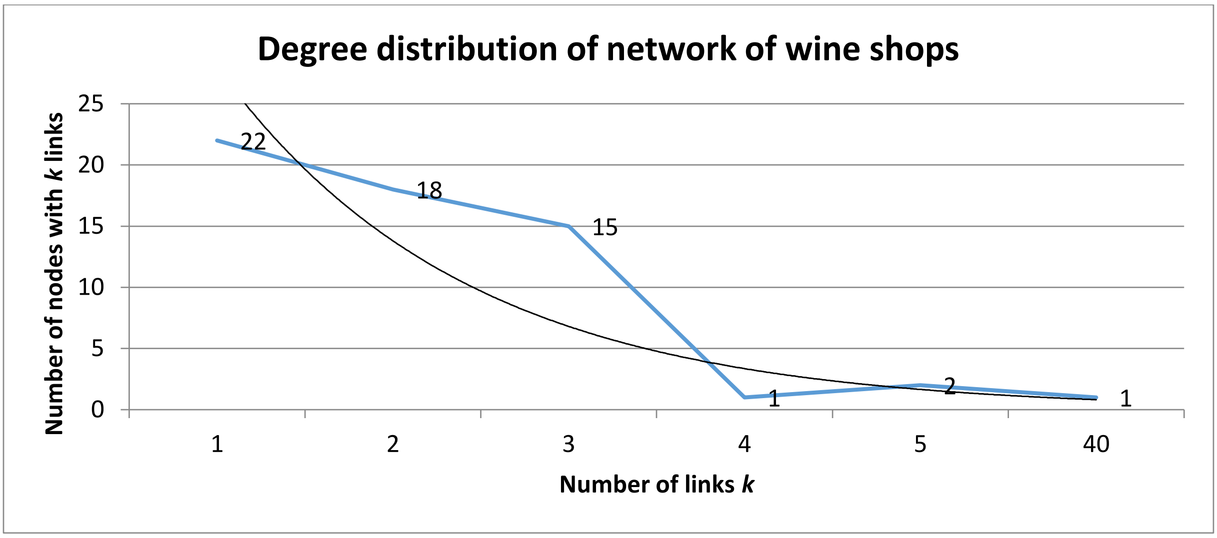
4.2.1. Degree distribution of NWS

NWS encompasses 41 vertices (*V*=41) and 55 undirected, unweighted edges (E=55):

$$\left(E = \frac{1}{2}\sum_{i=1}^{V} k_i\right) \tag{5}$$

The average degree of NWS $\left(\langle k \rangle = \frac{2E}{V}\right)$ is 2.68 meaning that a network player interacts with more than two enterprises. Notwithstanding, if results of degree distribution are analyzed (see Figure 2), a hub, the most connected vertex, can be recognized with 40 links. More precisely, a hub (focal firm) ties to every node but more than 53% of vertices possess only one connection ($k \ll \langle k \rangle \ll k_{hub}$); NWS is regarded to as a sparse network. Such a centralized network has been formed because rivals loathe each other because of their harmful earlier business experiences. Coopeting partners hardly ever interact with each other, they do not trust in each other but they believe in the hub that mediates among rivals and fills structural hole (i.e. it is a gap in the network among disconnected nodes. The hole is bridged by the hub to integrate the whole network) in NWS as well (Jóna-Tóth 2017, Jóna 2018).

Figure 2. Degree distribution of network of wine shops.



Resource: my calculation.

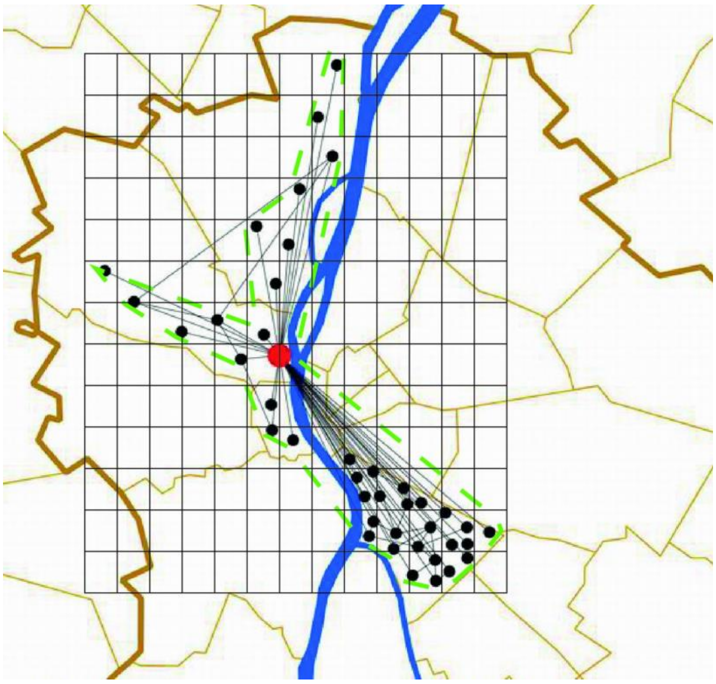
According to Figure 2, NWS does not have scale-free network property since degree exponent $\gamma = 0{,}707$ meaning belonging to the anomalous regime ($\gamma \leq 2$) (Luitz 2015, Newman 2005). Of course, the estimated value of γ has to be handled with caution because NWS<50 (Barabási 2016: 157). NWS, however, is not a scale-free network but operates effectively with anomalous topological edge modes. It implies that the robustness of NWS is relatively high against random targeting and attacking but it is low against consciously attacking. The special type of interconnectivity of NWS causes high vulnerability in the network.

In addition, the anomalous regime indicates that a giant component develops fastly in NWS; it links to every node and has a special, well-qualified ability to require connections. This super component appears as the main actor bridging among competitors and managing coopetitive activities in NWS.

4.2.2. Quadrat analysis of NWS

Firstly, grids embracing 13X13 matrixes are superimposed over the spatial layout of NWS and the number of events falling in each grid area is computed (see Figure 3 and Table 1). The longer side of the rectangle shaped cell is 1626 meter, and the shorter side is 1237 meter.

Figure 3. Quadrat analysis of NWS



Resource: my calculation.

Table 1 demonstrates that nodes are clustered extremely in the networked space, the degree and kind of clustered are computed with (2), (3) and (4) equations.

Table 1. Spatial pattern of NWS is visualized by quadrat analysis.

0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	1	1	0	0	0	0	0	0
1	0	0	0	0	1	0	0	0	0	0	0	0
0	1	1	0	1	1	0	0	0	0	0	0	0
0	0	0	0	1	**1**	0	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	1	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	3	3	1	0	0
0	0	0	0	0	0	0	0	2	2	2	4	1
0	0	0	0	0	0	0	0	0	0	3	2	0

Resource: my own calculation. The red number illustrates location of hub in the networked space.

Table 2. Quadrat counts and calculation of the variance for the NWS

Number of events *(a)*	**Number of quadrat (x)**	$\boldsymbol{a-\mu}$	$(\boldsymbol{a-\mu})^2$	$\boldsymbol{x(a-\mu)^2}$
0	141	-0.2426	0.0588547	8.2985127
1	20	0.7574	0.57365476	11.4730952
2	4	1.7574	3.08845476	12.35381904
3	3	2.7574	7.60325476	22.80976428
4	1	3.7574	14.11805476	14.11805476
Total	**169**			**69.05324598**

Source: my calculation. Mean cell count (μ): $\mu = \frac{V}{x}$; $\mu = \frac{41}{169} = 0.2426$. Variance: $\frac{69.05324598}{41} = 1.6842255$.
Variance/mean cell count (VMR): $\frac{1.6842255}{0.2426} = 6.942396$

Table 2 and belonging calculations report that $\mu = 0.2426$, the s^2=1.6842255, and VMR=6.942396. Since VMR is greater than 1 ($VMR \gg 1$), spatial patterns of NWS are regarded as a clustered, centralized and complex system but it is not a star graph.

Besides, the quadrat analysis permits to describe the physical distance between nodes. Empirical findings show that the average distance of linkages is 8.206 meters, the longest distance is 21.140 meters and the shortest distance is 364 meters in NWS.

Moreover, by applying quadrat analysis, spatial position of hub can be identified. Firstly, those grids must be found in which giant component exist. It can be obtained simply if we pinpoint and mark the cells in which the most connected vertex/vertices appear. By using this method, the physical position of hub could be stated in NWS locating in almost the center of a networked space (see Table 1). Interestingly, position of hub can be discovered relatively far from the groups of clustered network players, the average distance between the super component and southwestern clustered nodes are almost 4.5 km. The paper presupposed that numerous vertices would allocate close to the hub but the empirical measurement underscores that the intrinsic quality of the connection between the focal firm and network agents may not be depended on by physical proximity or distance. The spatial distance can be defeated by cognitive and social proximities that stem from relational capital of hub (Boschma 2005).

To summarized, the degree distribution points out that the hub ties to every competitor while rivals rarely connect to each other; the NWS possesses anomalous topological edge modes following dispersion pattern. The quadrat analysis, however, accentuates that the super-components of the NWS geographically and physically exist relatively far from clustered of vertices; the hub emerges in almost the center of networked space (see Table 1) meanwhile clustered network agents located on the periphery. Both edges and nodes are significantly clustered in NWS.

Conclusion

The paper develops and tests a complex model of network structure exploration with which architecture of real networks can be analyzed. This model perceives both structural properties of edges and vertices simultaneously to understand natures, functions and robustness of real networks. Why is it important fundamentally? It is because real networks can be stimulated or destroyed easily by supporting or attacking of the hub. Network structure explorations based on degree distribution are able to show whether hub exists in the network but unable to answer the question where the hub is in the networked space. By using CMNSE, however, both can be quantified easily.

More specifically, useful networks (societal, market-based economic, motorway, street, infrastructural, etc.) and harmful networks (terrorist, hackers, gossip, mafia, hoax, virus, etc.) can be distinguished. The first one should be developed and the second one must be destroyed. If useful network architecture is characterized with CMNSE, the relationships and physical location of hub and nodes can be managed, organized and incentivized consciously in order to strengthen the robustness of network functions and, as a result, the network can achieve purposes (Barabási 2016). It is enough if the hub is supported because the effects of supporting are allocated among nodes by the super-component that exists as an Achilles Heel in the network. By contrast, if structure of harmful network is quantified by CMNSE, it can also be controlled, targeted and attacked effectively. After mapping up patterns of connections and spatial position of the hub, harmful network can be targeted and destroyed and the whole network may collapse. If network structure is explored with CMNSE, it can be developed or ruined easily. More broadly, the degree of robustness and vulnerability of the real network (irrespective of it is useful or harmful) could be diminished or increased if the network topology is gauged by CMNSE.

The main limit of CMNSE is that only structural traits of two-dimensional networks can be described with it. Topologies of three-dimensional networks cannot be characterized by this model. Next task is that CMNSE has to be improved to scrutinize three-dimensional network architecture in the future.

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